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On the connection between spin and statistics for the massless spinor field*

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Abstract. In the case of a massless spinor field in 4D spacetime it is proved that both Fermi and Bose quantizations can be carried out. The Bose quantization of this field is demonstrated.

The aim of the present paper is to revise some aspects connected with the theory of a free massless spinor field and its quantization. As is well known, the massless Dirac equation is a combination of two Weyl equations, each of which is invariant under the proper Lorentz group. However, this combination is not unique. In fact, the two Weyl equations may be combined in a different way in which we obtain that the massless Dirac equation is equivalent to the Cauchy–Riemann–Fueter condition for the so-called Fueter quaternionic analyticity. The last form of this equation yields the possibility of seeing some interesting properties of the latter, the most singificant of which is the double character of the fields obeying the massless spinor theory.

In the present paper it will be proved that the spinor field may be considered in an alternative manner which allows its quantization with the help of Bose commutation relations. As is well known (Bogolubov and Schirkov 1973, Bogolubov *et al* 1975, Itzykson and Zuber 1980), in the usual approach the requirements of locality and energy positivity for the free spinor field lead to the Fermi quantization. In contrast, the Bose quantization does not fulfil these requirements. In this paper the opinion is that the more important of the latter is the energy positivity, because it depends on the type of commutation relations but not on the commutation functions.

We start with the action of the free quantum spinor field which we shall write in the form

$$A=\frac{\mathrm{i}}{2}\int :\bar{\psi}(x)\gamma^{\mu}\overleftrightarrow{\partial}_{\mu}\psi(x):\,\mathrm{d}^4x. \tag{1}$$

Here, as usual, we have denoted with $\overleftrightarrow{\partial}_{\mu}$ the following operation:

$$u\overleftrightarrow{\partial}_{\mu}v=u\partial_{\mu}v-\partial_{\mu}uv. \tag{2}$$

We assume that the kind of commutation relations which are obeyed by spinor field $\psi(x)$, is not specified. To define the normal product signified in (1) with $:\ldots:$ it is sufficient to assume that the fields $\psi(x)$, $\bar{\psi}(x)$ commute or anti-commute with each other under the sign $:\ldots:$.

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When the γ-matrices are in a diagonal γ^5-representation, instead of relation (1) we have

$$A=\frac{i}{2}\int :\varphi^*(x)(\overleftrightarrow{\partial}_0+\sigma_k\overleftrightarrow{\partial}_k)\varphi(x): \mathrm{d}^4x+\frac{i}{2}\int :\chi^*(x)(\overleftrightarrow{\partial}_0-\sigma_k\overleftrightarrow{\partial}_k)\chi(x): \mathrm{d}^4x \tag{3}$$

where σ_k are the Pauli matrices. To obtain the last equation we have given the field $\psi(x)$ as follows:

$$\psi(x)=\begin{pmatrix}\varphi(x)\\ \chi(x)\end{pmatrix} \tag{4}$$

where $\varphi(x)$ and $\chi(x)$ are two-component spinor fields which are transformed under the two non-equivalent spinor representation of the Lorentz group (Weyl representations). The asterisks denote Hermitian conjugation. Then we change the form of the secon term on the right-hand side of equation (3) in the following way:

$$\int :\chi^*(x)(\overleftrightarrow{\partial}_0-\sigma_k\overleftrightarrow{\partial}_k)\chi(x): \mathrm{d}^4x=\varkappa\int :\chi(x)\varepsilon(\overleftrightarrow{\partial}_0+\sigma_k\overleftrightarrow{\partial}_k)\varepsilon\chi^*(x): \mathrm{d}^4x. \tag{5}$$

Here $\varkappa$ takes two values: +1 if we have assumed that $\chi(x)$ and $\chi^*_{(x)}$ commute and −1 if we have assumed that they anti-commute. The matrix ε has the usual form

$$\varepsilon=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$$

with the property

$$\sigma_k^T=\varepsilon\sigma_k\varepsilon$$

which is used to obtain the right-hand side of equation (5). After we take the latter into account we can obtain a new expression for action A

$$A=\frac{i}{2}\int \mathrm{Tr}:\tilde{\phi}(x)(\overleftrightarrow{\partial}_0+\sigma_k\overleftrightarrow{\partial}_k)\phi(x): \mathrm{d}^4x. \tag{6}$$

$\phi(x)$ and $\tilde{\phi}(x)$ are the 2×2 matrix-valued fields which are expressed through the two-component fields $\varphi_\alpha(x)$, $\chi_\alpha(x)$ ($\alpha=1,2$) and their conjugation. In particular, one possible expression for these fields is the following:

$$\phi(x)=\begin{pmatrix}\varphi_1(x) & \chi_2^*(x)\\ \varphi_2(x) & -\chi_1^*(x)\end{pmatrix} \qquad \tilde{\phi}(x)=\begin{pmatrix}\varphi_1^*(x) & \varphi_2^*(x)\\ -\varkappa\chi_2(x) & \varkappa\chi_1(x)\end{pmatrix}. \tag{7}$$

In general these expressions are not defined uniquely because of the existence of a special gauge, with respect to which action (6) is invariant. This gauge has the following form:

$$\phi'(x)=\phi(x)A \qquad \text{and} \qquad \tilde{\phi}'(x)=A^{-1}\tilde{\phi}(x) \tag{8}$$

where A is an arbitrary non-degenerated constant complex matrix. Because of this invariance, the only statement we can make is that the fields $\phi(x)$ and $\tilde{\phi}(x)$ in general are not conjugate to each other. However, in the case when $\varkappa=-1$ there exists a gauge frame in which $\tilde{\phi}(x)$ coincides with $\phi^*(x)$ (i.e. the Hermitian conjugation). In the case when $\varkappa=1$ (i.e. the fields commute under the normal product sign) the analogous frame does not exist at all. Indeed, the reality condition for action (6) in particular leads to the following relation:

$$\phi^*(x)=s\tilde{\phi}(x) \tag{9}$$

where s is an arbitrary but fixed 2×2 matrix which must be Hermitian. In case (7) this is σ_0 (the unit matrix) if $\varkappa=-1$ and σ_3 (the third Pauli matrix) if $\varkappa=1$.

In general, the gauge transformations change s as follows:

$$s'=A^*sA \tag{10}$$

where s' is the same matrix in the new gauge. From equation (10) we can see that there exist two general ways for defining the field $\tilde{\phi}(x)$ which do not mix. Each of them is formed of matrices belonging to two different orbits of transformation (10) (i.e. of the group $\mathrm{GL}(2, C)$). The first is that which contains the unit matrix and the second is that which contains σ_3. It is easy to prove that these two matrices create, with the help of transformation (10), two non-intercepted orbits—a 'time-like' one and a 'space-like' one respectively. Furthermore, this gauge does not allow unique definition of the Lorentz structure of the fields considered. There exist three different ways to define the Lorentz transformations of the fields $\phi(x)$ and $\tilde{\phi}(x)$ that leave the action (6) invariant:

$$\begin{array}{ll}\phi(x)\to S(g)\phi(x_\nu\Lambda^\nu_\mu(g)) & \tilde{\phi}(x)\to\tilde{\phi}(x_\nu\Lambda^\nu_\mu(g))S^*(g)\\ \phi(x)\to S(g)\phi(x_\nu\Lambda^\nu_\mu(g))S^*(g) & \tilde{\phi}(x)\to S^{*-1}(g)\tilde{\phi}(x_\nu\Lambda^\nu_\mu(g))S^*(g)\\ \phi(x)\to S(g)\phi(x_\nu\Lambda^\nu_\mu(g))S^{-1}(g) & \tilde{\phi}(x)\to S(g)\tilde{\phi}(x_\nu\Lambda^\nu_\mu(g))S^*(g)\end{array} \tag{11}$$

where $S(g)$ belongs to the fundamental representation $(\frac{1}{2},0)$ of $\mathrm{SL}(2, C)$ ($S^*(g)$ belongs to the representation $(0,\frac{1}{2})$ of the same group and $\Lambda^\nu_\mu(g)$ belongs to the four-component vector representation of the Lorentz group ($g\in \mathrm{SL}(2, c)$ respectively).

Only the first of these transformations defines $\phi(x)$ and $\tilde{\phi}(x)$ as spinor fields. The others define them as a vector field or a field belonging to a reducible representation which consists of a scalar and a self-dual antisymmetric tensor field. Therefore, in both these cases the fields $\phi(x)$ and $\phi(x)$, describing a massless spinor particle, belong to 'very' different representations of the Lorentz group, which is here considered to be most unexpected. Recall that in the usual case $\phi(x)$ and $\tilde{\phi}(x)$ also belong to different representations but they are conjugate to each other.

The non-uniqueness pointed out in the definition and description of the massless Dirac field leads to several non-standard results in this theory. The most important is the possibility of quantizing this field with the help of Bose commutation relations. This statement is proved in the following paragraphs.

First, we must obtain a suitable solution of the equations of motion following from action (6):

$$(\partial_0+\sigma_k\partial_k)\phi(x)=0 \tag{12}$$

$$\tilde{\phi}(x)(\overleftarrow{\partial}_0+\sigma_k\overleftarrow{\partial}_k)=0 \qquad (u\overleftarrow{\partial}\equiv\partial u). \tag{13}$$

Remark. Let us consider the first of these equations. This equation in the Euclidean formulation (i.e. when $X_4=\mathrm{i}X_0$) coincides with the so-called Fueter analyticity conditions (Fueter 1935, 1936, Sudbery 1979) for the quaternionic functions $\phi(x)$. Here we are dealing with a formal analogy of the free Maxwell equations which also coincide with these conditions (Weingarten 1973). The connections between the solutions of the free Maxwell equations and those of the free massless Dirac equations manifested in the paper by Fushchich *et al* (1991) are a simple consequence of the fact that these two equations coincide with each other in the quaterionic form. In a sense the results of the present paper extend this formal connection between the two fundamental fields.

In the momentum representation each of equations (12) and (13) has four linear-independent solutions which form a complete set of orthonormal eigenvectors of the energy. We shall denote these vectors as $u^{\pm}(\boldsymbol{p})$, $v^{\pm}(\boldsymbol{p})$ for the first equation

$$(\pm|\boldsymbol{p}|-\sigma_k p_k)u^{\pm}(\boldsymbol{p})=(\pm|\boldsymbol{p}|+\sigma_k p_k)v^{\pm}(\boldsymbol{p})=0 \tag{14}$$

and as $\tilde{u}^{\pm}(\boldsymbol{p})$, $\tilde{v}^{\pm}(\boldsymbol{p})$ for the second equation

$$\tilde{u}^{\pm}(\boldsymbol{p})(\pm|\boldsymbol{p}|-\sigma_k p_k)=\tilde{v}^{\pm}(\boldsymbol{p})(\pm|\boldsymbol{p}|+\sigma_k p_k)=0. \tag{15}$$

In the case under consideration $\tilde{u}$ and $\tilde{v}$ may not be conjugate to u and v respectively, reflecting the orthonormality condition of these vectors. As is well known, the latter must be covariant under the action of the corresponding representation of the Poincaré group. Here the scalar product corresponding to the definition of action (6) has the following form:

$$(a, b)=\mathrm{Tr}\,\tilde{a}b \tag{16}$$

for any two 2×2 matrix-valued vectors of the type of the eigenvectors u and v mentioned above. This scalar product does not distinguish the three different transformations (11) because of its invariance with respect to transformations (8). That is why the subsequent results about orthonormality properties of the eigenvectors are valid in all three cases.

Because of the fact that in our case for each eigenvalue of the energy $p_0=\pm|\boldsymbol{p}|$ we have two linearly independent eigenvectors, $u^{\pm}(\boldsymbol{p})$ and $v^{\mp}(\boldsymbol{p})$ respectively, we assume that

$$(u^A(\boldsymbol{p}), v^B(\boldsymbol{p}))=(v^A(\boldsymbol{p}), u^B(\boldsymbol{p}))=0 \qquad A, B=\pm. \tag{17}$$

The quantities

$$(u^A(\boldsymbol{p}), u^B(\boldsymbol{p})) \tag{18}$$

and analogously

$$(v^A(\boldsymbol{p}), v^B(\boldsymbol{p})) \tag{19}$$

(as well as the quantities in equation (17)) change under the action of transformations (11) as second-rank tensors with respect to the maximal compact subgroup O(2) of the Wigner group E(2) of the Poincaré group (Fushchich and Nikitin 1987). We use the quantities (18) and (19) to define the normalization condition which must be invariant under the action of transformations (11) (see the appendix). To reach this object we must identify tensors (18) and (19) with the invariant ones of the group O(2). By analogy with the massive case we can choose as such a tensor the unit one δ_{AB}. However, the group O(2) is Abelian and δ_{AB} is not the only invariant tensor. The generator $(\sigma_3)_{AB}$ has the same property, which means that we can normalize our eigenvectors in two different ways—with either the help of δ_{AB} or $(\sigma_3)_{AB}$. The first possibility is used in the Fermi quantization of the Dirac field when $\tilde{u}$ and $\tilde{v}$ coincide with u^* and v^* respectively. In our case, when the latter is not fulfilled, we must use the second possibility:

$$(u^A(\boldsymbol{p}), u^B(\boldsymbol{p}))\equiv\mathrm{Tr}[\tilde{u}^A(\boldsymbol{p})\cdot u^B(\boldsymbol{p})]=(\sigma_3)_{AB} \tag{20}$$

and analogously

$$(v^A(\boldsymbol{p}), v^B(\boldsymbol{p}))\equiv\mathrm{Tr}[\tilde{v}^A(\boldsymbol{p})\cdot v^B(\boldsymbol{p})]=-(\sigma_3)_{AB}$$

$$((\sigma_3)_{++}=-(\sigma_3)_{--}=1 \qquad (\sigma_3)_{+-}=(\sigma_3)_{-+}=0).$$

To construct the concrete form of these orthonormal eigenvectors we can use the unitary matrix $U(\boldsymbol{p})$ defined by the following equality:

$$U(\boldsymbol{p})\sigma_3 U^*(\boldsymbol{p}) = \frac{p_k\sigma_k}{|\boldsymbol{p}|} \qquad (|\boldsymbol{p}| = \sqrt{p_1^2+p_2^2+p_3^2}). \tag{21}$$

Then we have

$$u^{\pm}(\boldsymbol{p}) = \tfrac{1}{2}\, U(\pm\boldsymbol{p})(1+\sigma_3) \equiv \frac{|\boldsymbol{p}| \pm \sigma_k p_k}{2|\boldsymbol{p}|}\, U(\pm\boldsymbol{p})$$

$$v^{\pm}(\boldsymbol{p}) = \tfrac{1}{2}\, U(\mp\boldsymbol{p})\sigma_1(1-\sigma_3) \equiv \frac{|\boldsymbol{p}| \mp \sigma_k p_k}{2|\boldsymbol{p}|}\, U(\mp\boldsymbol{p})\sigma_1 \tag{22}$$

and

$$\tilde{u}^{\pm}(\boldsymbol{p}) = \pm(u^{\pm}(\boldsymbol{p}))^* \qquad \tilde{v}^{\pm}(\boldsymbol{p}) = \mp(v^{\pm}(\boldsymbol{p}))^*. \tag{23}$$

Now we can write the general solution of equations (12) and (13). Without entering into details, these solutions have the following form:

$$\phi(x) = \frac{1}{(2\pi)^{3/2}} \int \{[a_1^+(\boldsymbol{p})u^+(\boldsymbol{p}) + a_2^+(\boldsymbol{p})v^-(\boldsymbol{p})]\exp[\mathrm{i}|\boldsymbol{p}|x_0 - \mathrm{i}\boldsymbol{px}]$$
$$+ [b_1^-(-\boldsymbol{p})v^+(\boldsymbol{p}) + b_2^-(-\boldsymbol{p})u^-(\boldsymbol{p})]\exp[-\mathrm{i}|\boldsymbol{p}|x_0 - \mathrm{i}\boldsymbol{px}]\}\,\mathrm{d}^3p \tag{24}$$

and

$$\tilde{\phi}(x) = \frac{1}{(2\pi)^{3/2}} \int \{[a_1^-(\boldsymbol{p})\tilde{u}^+(\boldsymbol{p}) + a_2^-(\boldsymbol{p})\tilde{v}^-(\boldsymbol{p})]\exp[-\mathrm{i}|\boldsymbol{p}|x_0 + \mathrm{i}\boldsymbol{px}]$$
$$+ [b_1^+(-\boldsymbol{p})\tilde{v}^+(\boldsymbol{p}) + b_2^+(-\boldsymbol{p})\tilde{u}^-(\boldsymbol{p})]\exp[\mathrm{i}|\boldsymbol{p}|x_0 + \mathrm{i}\boldsymbol{px}]\}\,\mathrm{d}^3p \tag{25}$$

where $a_\alpha^+(\boldsymbol{p})$, $b_\alpha^+(\boldsymbol{p})$, $a_\alpha^-(\boldsymbol{p})$, $b_\alpha^-(\boldsymbol{p})$ $(\alpha = 1, 2)$ are the cretation and annihilation operators and

$$(a_\alpha^{\pm}(\boldsymbol{p}))^* = a_\alpha^{\mp}(\boldsymbol{p}) \qquad (b_\alpha^{\pm}(\boldsymbol{p}))^* = b_\alpha^{\mp}(\boldsymbol{p}). \tag{26}$$

From the Noether theorem we obtain the energy-momentum tensor:

$$T_{\mu\nu}(x) = \frac{\mathrm{i}}{2}\int \mathrm{Tr} : \tilde{\phi}(x)\sigma_\mu \overleftrightarrow{\partial}_\nu \phi(x) : \mathrm{d}^3x. \tag{27}$$

The momentum P_μ has the form

$$P_\mu = \frac{\mathrm{i}}{2}\int \mathrm{Tr} : \tilde{\phi}(x)\overleftrightarrow{\partial}_\mu \phi(x) : \mathrm{d}^3x. \tag{28}$$

Then we must substitute $\phi(x)$ and $\tilde{\phi}(x)$ in equation (28) with their expressions from equations (24) and (25).

After simple but very long calculations we obtain the following form for the momentum operators:

$$P_\mu = \int p_\mu : [b_\alpha^+(\boldsymbol{p})b_\alpha^-(\boldsymbol{p}) + a_\alpha^-(\boldsymbol{p})a_\alpha^+(\boldsymbol{p})] : \mathrm{d}^3p. \tag{29}$$

We see that the sign of the second term on the right-hand side of equation (25) is opposite to that in the usual case and shows us that we must quantize with the help of Bose commutation relations instead of Fermi ones. From the Hamiltonian equations

$$\mathrm{i}\partial_0\phi(x) = [P_0, \phi(x)]_-$$

we define the following commutation relations between the operators $a_\alpha^+(\boldsymbol{p})$, $a_\alpha^-(\boldsymbol{p})$, $b_\alpha^+(\boldsymbol{p})$, $b_\alpha^-(\boldsymbol{p})$:

$$[a_\alpha^-(\boldsymbol{p}), a_\beta^+(\boldsymbol{q})]_- = \delta_{\alpha\beta}\delta^3(\boldsymbol{p}-\boldsymbol{q})$$
$$[b_\alpha^-(\boldsymbol{p}), b_\beta^+(\boldsymbol{q})]_- = \delta_{\alpha\beta}\delta^3(\boldsymbol{p}-\boldsymbol{q}) \tag{30}$$

all other commutators vanish. The same commutators can also be obtained in the canonical approach. Then we can calculate the commutation relations for the fields $\phi(x)$ and $\tilde{\phi}(y)$. As a result, we find that

$$[\phi_{\alpha\beta}(x), \tilde{\phi}_{\gamma\delta}(y)]_- = (\partial_0 - \sigma_k\partial_k)_{\alpha\delta} D_0(x-y)\delta_{\beta\gamma} \tag{31}$$

where the function $D_0(z)$ is the Pauli–Jordan function for the massless scalar field:

$$D_0(z) = \frac{\mathrm{i}}{(2\pi)^3}\int \varepsilon(p_0)\delta(p^2)\,\mathrm{e}^{\mathrm{i}pz}\,\mathrm{d}^4p. \tag{32}$$

All other commutators are equal to zero. Equality (31) is a proof for the locality of the Bose fields $\phi(x)$ and $\tilde{\phi}(x)$.

Using the thus constructed Bose-quantized theory for the spinor field we can also calculate some other quantities. First, this may be the operator of the particle charge. It reads

$$\rho = \int \mathrm{Tr} : \tilde{\phi}(x)\phi(x) : \mathrm{d}^3x$$
$$= \int [a_\alpha^+(\boldsymbol{p})a_\alpha^-(\boldsymbol{p}) - b_\alpha^+(\boldsymbol{p})b_\alpha^-(\boldsymbol{p})]\,\mathrm{d}^3p. \tag{33}$$

Another quantity is the two-point function $\langle 0|\tilde{\phi}_{\alpha\beta}(x)\phi_{\gamma\delta}(y)|0\rangle$. To calculate the latter we choose the operators $a_\alpha^-(\boldsymbol{p})$ and $b_\beta^-(\boldsymbol{p})$ as anninilation operators. Then we have

$$\langle 0|\tilde{\phi}_{\alpha\beta}(x), \phi_{\gamma\delta}(y)|0\rangle = (\partial_0 - \sigma_k\partial_k)_{\gamma\beta} D_0^-(x-y)\delta_{\alpha\delta} \tag{34}$$

where the function $D_0^-(z)$ is the negative frequency part of the Pauli–Jordan function $D_0(z)$:

$$D_0^-(z) = \frac{\mathrm{i}}{(2\pi)^3}\int \frac{\mathrm{d}^3p}{2p_0}\mathrm{e}^{-\mathrm{i}px} \qquad (p_0 = |\boldsymbol{p}|). \tag{35}$$

Finally, the spin operator will be considered in the present case. As is well known, the spin operator of the spinor field is not a time-independent quantity. However, in the case of a massless spinor field the preserving quantity is the helicity

$$\lambda = \frac{P_k}{P_0} S_k \tag{36}$$

where S_k are the spin parts of the first three generators of the Lorentz group which correspond to the rotation subgroup.

The helicity has a meaning only with the momentum-defined states. That is why the operators S_k must be chosen in the following form:

$$S_k = -\tfrac{1}{2}\mathrm{Tr} : \tilde{\phi}_p(x)\sigma_k\phi_p(x) : \tag{37}$$

where $\tilde{\phi}_p(x)$ and $\phi_p(x)$ are the momentum-defined solutions of equations (12) and (13) (expressions in the integrands in equations (24) and (25)).

Remark. Because of the existence of three different Lorentz transformations (11) which leave the present theory invariant, three different helicity operators can be defined for the fields. Here the helicity corresponding to the first transformation only will be analysed. The two others can be easily obtained if it is noted that the operators S_k in the second and third cases of equation (11) have the form

$$S_k = -\tfrac{1}{2}\,\mathrm{Tr} : \tilde{\phi}_p(x)[\sigma_k, \phi_p(x)]_- : .$$

The formal coincidence of these operators follows from the identity between rotation subgroup structure of the transformations considered (although they belong to different Lorentz representations).

It can be seen from equation (36) that in the light-cone frame (i.e. $p_1 = p_2 = 0$) the expression for the helicity takes the simplest form:

$$\lambda = S_3 .$$

With the help of equation (37) λ can be obtained in the following form:

$$\lambda = \tfrac{1}{2}\{[b_1^+(-p_3)b_1^-(-p_3) + b_2^+(-p_3)b_2^-(-p_3)]\theta(-p_3) - [a_1^+(p_3)a_1^-(p_3) + a_2^+(p_3)a_2^-(p_3)]\theta(p_3)$$

$\theta(p)$ being the step function.

It can be seen that the term proportional to $\theta(-p)$ has helicity $-\frac{1}{2}$ and the term with $\theta(p)$-helicity $+\frac{1}{2}$.

The quantization of the spinor field as a Bose field gives the opportunity of looking at the neutrino from another viewpoint. First, let us note that for this type of quantization the most important requirement is that the neutrino is massless. This means that the type of statistics satisfied by the neutrino leads to definite conclusions about its mass. Moreover, with the Bose neutrino field it is possible to write new interaction terms in the Lagrangian as

$$\psi_L^* \phi \psi_L \qquad \psi_R^* \phi \psi_R \qquad \psi_L^* \tilde{\phi} \psi_L \ldots$$

and so on, where ψ_L and ψ_R are respectively the left- and right-handed components of any massive spinor field (e.g. the electron field). It may be assumed that such interactions must be weaker than the weak interaction and that the manifestation of neutrino as a Bose or a Fermi particle depends on the character of the interactions.

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Appendix

The action of the first transformation (11) on the fields $\phi(x)$ and $\tilde{\phi}(x)$ from equation (11) on the fields $\phi(x)$ and $\tilde{\phi}(x)$ from equations (24) and (25) changes the eigenvectors as follows:

$$u^{\pm}(p') \rightarrow \sqrt{\frac{|p|}{|p'|}}\, S(g) u^{\pm}(\Lambda^{\rho}\sigma(g^{-1})p'^{\sigma}) = e^{i\alpha(g,p)} u^{\pm}(p') \tag{A.1}$$

$$\tilde{u}^{\pm}(p') \rightarrow \sqrt{\frac{|p|}{|p'|}}\, \tilde{u}^{\pm}(\Lambda^{\rho}\sigma(g^{-1})p'^{\sigma}) S^*(g) = e^{-i\alpha(g,p)} \tilde{u}^{\pm}(p') \tag{A.2}$$

where $p^{\mu}=\Lambda^{\mu}_{\rho}(g^{-1})p'^{\rho}$, $p^2=0$ and $\alpha(g, p)$ is a phase depending on the group element g and on the momentum p^{μ} which is a concrete form of no importance. The equalities in the expressions (A.1) and (A.2) can be obtained with the help of direct calculations, using definitions (23) and (24) of the vectors $u^{\pm}$ and $\tilde{u}^{\pm}$.

From equations (A.1) and (A.2) the invariance of the normalization condition (20) becomes obvious.

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